

The phenomenology of rare and semileptonic B decays¹

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We summarize recent progress in the theory of exclusive rare and semileptonic B decays, focusing on model-independent results. The heavy-to-light form factors parameterizing these decays admit a model-independent description in two distinct kinematical regions. In the large-energy limit of an energetic light meson, the Soft-Collinear Effective Theory can be used to prove factorization formulas for the form factors. We present factorization formulas for all $B \rightarrow P, V$ form factors at leading order in Λ/m_b . Near the zero-recoil point, Heavy Quark Effective Theory gives useful relations among the form factors of different currents.

1 Introduction

The exclusive B decays are, in many ways, unique probes of the Standard Model and its extensions. The semileptonic B decays to charmless states can give information about $|V_{ub}|$, while the exclusive radiative decays $B \rightarrow \rho(K^*)\gamma$ decays can be used to extract $|V_{td}|$. In addition, many of these decays are flavor changing neutral current processes, which proceed only through loops. Therefore they are sensitive to the presence of new nonstandard particles running in the loop, and can be used to search for new physics effects [1].

Experimental data on these decays is becoming available, including not only branching ratios, but also spectrum shapes in semileptonic decays [2]. Interpreting this data for the purpose of extracting CKM parameters and in searching for New Physics effects requires good control over the Standard Model description of these decays. Many computations of these form factors are available, using methods as varied as quark models, QCD sum rules (see [3] for a recent review) and lattice QCD [4]. We will focus here on recent model independent results.

There are two kinematical regions where model independent results can be established, corresponding to the two kinematical limits of: a) slow and b) energetic final light hadrons. They are most naturally discussed in terms of two effective theories: a) Heavy Quark Effective

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Theory (HQET) and b) Soft-Collinear Effective Theory (SCET). Rather than following the historical order of events, we will discuss these two types of predictions starting from the respective effective theory describing each of these two situations. The large energy limit will be discussed in Sec. II and the case of the low recoil form factors is covered in Sec. III. An Appendix contains a summary of the factorization formulas for the $B \rightarrow P, V$ form factors contributing to rare and semileptonic B decays.

2 Large energy limit and the SCET

The Soft-Collinear Effective Theory (SCET) was proposed in [5] as a systematic framework for the study of processes involving energetic quarks and gluons, and was discussed in detail in another talk at this Conference [6]. In this talk we summarize only a few points strictly necessary for the discussion of the form factors.

The SCET separates the contributions from the different energy scales relevant in a physical problem. This is done by introducing fields with well-defined momentum scaling, corresponding to the modes relevant for reproducing the IR of the full theory. These modes include i) collinear quarks ξ_n and gluons A_n with momenta $p_c \sim Q(\lambda^2, 1, \lambda)$, ii) usoft quarks q and gluons A_μ with momenta $p_{us} \sim Q\lambda^2$ and iii) soft modes q_s, A_s^μ with momenta $p_s \sim Q\lambda$. The definition of the expansion parameter λ depends on the specific problem being studied. We use here and below light-cone component notation $p = (n \cdot p, \bar{n} \cdot p, p_\perp)$, defined in terms of light cone unit vectors $n^2 = \bar{n}^2 = 0, n \cdot \bar{n} = 2$.

The couplings of the effective theory fields are described by the SCET Lagrangian. In a theory containing only usoft and collinear modes, these couplings can be written as²

$$\mathcal{L}_{SCET} = \mathcal{L}_{\xi\xi} + \mathcal{L}_{cg} + \mathcal{L}_{q\xi}, \quad (1)$$

where the first two terms describe couplings of collinear fields to each other ($\mathcal{L}_{\xi,cg}$) [5] and usoft-collinear interactions ($\mathcal{L}_{q\xi}$) [9, 10], respectively. They can be expanded in λ as

$$\mathcal{L}_{\xi\xi} = \mathcal{L}_{\xi\xi}^{(0)} + \mathcal{L}_{\xi\xi}^{(1)} + \dots, \quad \mathcal{L}_{q\xi} = \mathcal{L}_{q\xi}^{(1)} + \mathcal{L}_{q\xi}^{(2)} + \dots, \quad (2)$$

where the leading order collinear quark Lagrangian is (with $iD_{us}^\mu = i\partial^\mu + gA_{us}^\mu$)

$$\mathcal{L}_{\xi\xi}^{(0)} = \bar{\xi}_n \left\{ n \cdot iD_{us} + gn \cdot A_n + i \not{D}_{\perp c} \frac{1}{\bar{n} \cdot iD_c} i \not{D}_{\perp c} \right\} \frac{\not{n}}{2} \xi_n \quad (3)$$

The explicit form of \mathcal{L}_{cg} can be found in Ref. [7]. Note the fact that the usoft-collinear Lagrangian $\mathcal{L}_{q\xi}$ starts at subleading order with terms of $O(\lambda)$. The weak current $\bar{q}\Gamma b$ is analogously matched onto SCET operators as [5, 8, 9, 10]

$$\bar{q}\Gamma b = \int d\omega C_0(\omega) J_0(\omega) + \int d\omega C_{1a}(\omega) J_{1a}(\omega) + \int d\omega_1 d\omega_2 C_{1b}(\omega_1, \omega_2) J_{1b}(\omega_1, \omega_2) + \dots \quad (4)$$

where the ellipses denote operators suppressed by $O(\lambda^2)$. The most general form of these operators is given in Ref. [10] for all allowed Dirac structures Γ .

²The complete case containing also soft fields is discussed in Refs. [7, 23, 17, 20].

An important property of the effective theory is ultrasoft-collinear factorization at leading order in λ . Since the usoft gluons couple to collinears only through the first term in Ref. (3), their effects can be absorbed at this order into a Wilson line $Y_n[n \cdot A]$ by performing a field redefinition of the collinear fields [7]

$$\xi_n = Y_n[n \cdot A_{\text{us}}] \xi_n^{(0)}, \quad A_n^\mu = Y_n A_n^{(0)\mu} Y_n^\dagger, \quad Y_n[n \cdot A] \equiv P \exp \left(ig \int_{-\infty}^0 ds n \cdot A_{\text{us}}(ns) \right). \quad (5)$$

The new collinear fields $\xi_n^{(0)}$ and $A_n^{(0)}$ do not couple to the usoft gluon field A_{us} , which now appears only through the Wilson line $Y[n \cdot A_{\text{us}}]$.

2.1 Factorization in $B \rightarrow \gamma \ell \nu$

The simplest exclusive heavy meson process which can be described in this framework is the leptonic radiative decay $B^\pm \rightarrow \gamma \ell^\pm \nu$. This decay proceeds through weak annihilation of the B constituent quarks and does not suffer from the chirality suppression affecting the pure leptonic decay $B \rightarrow \ell \nu$. Model dependent estimates [12] suggest branching ratios for this mode of the order of $\sim 10^{-6}$, which should be within the reach of the B factories.

We will be interested in the kinematical region where the photon energy E_γ is much larger than Λ_{QCD} , but can be smaller or comparable with the heavy quark mass m_b . In this region there are three relevant energy scales: the hard scale Q , with $Q = \{m_b, E_\gamma\}$, the collinear scale $p_c^2 \sim Q\Lambda$, and the soft scale $p_s^2 \sim \Lambda^2$. The soft scale is introduced by the typical momenta of the soft spectators in the B meson, and the collinear scale gives the typical virtuality of a spectator quark after being struck by the energetic photon $p_c = p_{sp} + q$. Finally, the hard scale is associated with hard gluons with virtualities of the order of the heavy quark mass.

Using SCET methods, a factorization theorem was proved in [11] to all orders in α_s , expressing the form factors for this mode at leading order in Λ/Q as (with $Q = \{m_b, E_\gamma\}$)

$$f_{V,A}(E_\gamma) = \frac{Q_q f_B m_B}{2E_\gamma} C_{V,A}(E_\gamma, \mu) \int dk_+ \frac{1}{k_+} J(E_\gamma k_+, \mu) \phi_B^+(k_+, \mu) \quad (6)$$

The three factors in this formula are connected with the three different scales in this problem: the Wilson coefficients $C_{V,A}$ appear in the matching of the heavy-to-light currents $\bar{u} \gamma_\mu (\gamma_5) b$ onto SCET operators, the jet function $J(E_\gamma k_+, \mu) = 1 + O(\alpha_s(p_c^2))$ accounts for effects associated with the collinear scale, and the B meson light-cone wave function (normalized as $\int dk_+ \phi_B^+(k_+) = 1$) accounts for fluctuations over the scale of the soft modes. Factorization in $B \rightarrow \gamma e \nu$ was also studied in [12, 13].

We mention here a few implications of the factorization formula Eq. (6). At tree level in matching at the scale $Q\Lambda$, it predicts that the form factors in $B \rightarrow \gamma \ell \nu$ are proportional to the first inverse momentum of the B light-cone wave function $\langle k_+^{-1} \rangle$. The same moment appears in many other factorization formulas for B meson decays. Therefore, measurements of the photon spectrum in $B \rightarrow \gamma \ell \nu$ could provide a model-independent extraction of this parameter. Second, all LO form factors determining $B \rightarrow \gamma \ell \nu$, $B_s \rightarrow \gamma \ell^+ \ell^-$ and $B \rightarrow \gamma \gamma$ decays are given by one single nonperturbative integral over the B wave functions [11, 13]. Therefore their ratios can be computed in terms of the Wilson coefficients $C_{V,A,T}$ which have expansions in $\alpha_s(Q)$ and contain double Sudakov logs. Finally, the corrections to the factorization formula Eq. (6) are suppressed by Λ/Q and come from matrix elements of power suppressed operators in the SCET.

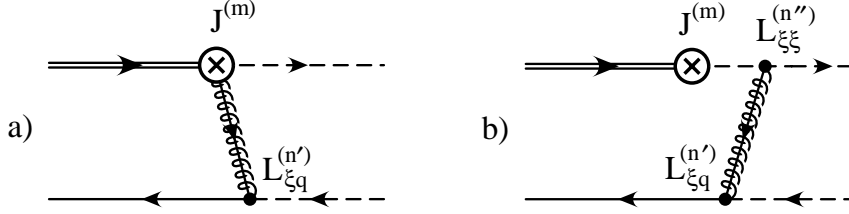


Figure 1: Tree level contributions to the T -products (7) in SCET_I .

2.2 Factorization for heavy-light form factors

We consider next the case of the heavy-to-light form factor, relevant for the semileptonic decays $B \rightarrow \pi(\rho)\ell\nu$, or the rare radiative decays $B \rightarrow K^*\gamma, K^*\ell^+\ell^-$. The dynamics for this case is more complicated than for the leptonic radiative decay due to the presence of the collinear partons in the final state light meson.

Although it had been known for a long time that exclusive hadronic form factors admit a systematic expansion in perturbative QCD at large momentum transfer Q [14], the extension of this approach beyond leading order in Λ/Q has been a difficult task. At subleading order one quickly encounters difficulties connected with soft parton configurations, corresponding to situations where one of the partons in a given hadron carries most of the hadron momentum. In heavy-to-light problems, such effects appear already at leading order in Λ/m_b . In the standard hard scattering analysis, they lead to unintegrable singularities in the hard scattering kernels [15] which at $O(\alpha_s)$ can be absorbed into soft form factors ζ [16].

The heavy-light form factor was recently studied in [17] using the SCET, where a factorization theorem at leading order in Λ/Q was established. The main points are:

- There are two relevant perturbative scales in this problem: Q and $\sqrt{\Lambda Q}$, where $Q = \{m_b, E_\pi\}$. The effects associated with these two scales can be included using a two-step matching: $\text{QCD} \rightarrow \text{SCET}_\text{I} \rightarrow \text{SCET}_\text{II}$. Here SCET_I contains collinear modes with $p_c^2 \sim Q\Lambda$ and usoft modes with $p_{us}^2 \sim \Lambda^2$, and SCET_II includes collinear and soft modes with $p_c^2 = p_s^2 = \Lambda^2$.
- The light meson state is purely collinear, and couples to the soft B meson state through the weak currents J given in Eq. (4), and the usoft-collinear Lagrangian $\mathcal{L}_{q\xi}$ in Eq. (2). In SCET_I , the weak current contributing to the heavy-to-light form factor is matched at LO onto the T -products

$$\begin{aligned} T_1^F &= T[J^{(1a)}, i\mathcal{L}_{\xi q}^{(1)}], & T_2^F &= T[J^{(1b)}, i\mathcal{L}_{\xi q}^{(1)}], \\ T^{NF} &= T[J^{(0)}, i\mathcal{L}_{\xi\xi}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}] + T[J^{(0)}, i\mathcal{L}_{\xi q}^{(2)}] + T[J^{(0)}, i\mathcal{L}_{c\xi}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}] + T[J^{(0)}, i\mathcal{L}_{\xi q}^{(1)}]. \end{aligned} \quad (7)$$

These contributions are shown in a graphical form in Fig. 1.

- After usoft-collinear factorization Eq. (5), the T -products $T_{1,2}^F$ factor, and can be matched directly onto SCET_II operators. Their matrix elements give the ‘factorizable’ contribution. On the other hand, the NF T -products all involve J_0 , and their matrix elements are

parameterized in terms of three ‘soft’ form factors, appearing in the same combinations as derived in [21].

In this way one finds that at leading order in Λ/Q , the heavy-to-light $B \rightarrow M$ form factors can be represented by a factorization formula written schematically as

$$f_i(q^2) = C_{ij}(Q)\zeta_j^M(Q\Lambda, \Lambda^2) + \int dx dz dk_+ C_{ij}(z, Q) J_j(z, x, k_+) \phi_B^+(k_+) \phi_j^M(x) \quad (8)$$

Here C_{ij} are Wilson coefficients of SCET_I operators, and $J_j(z, x, k_+)$ are jet functions appearing as Wilson coefficients in matching onto SCET_{II} operators. Both these quantities are computable in perturbation theory. The nonperturbative effects in Eq. (8) occur in the form factors ζ_i , and the light-cone wave functions of the B meson and of the light meson $\phi_B^+(k_+), \phi_i(x)$. In the Appendix we give the explicit form of the factorization formulas for all $B \rightarrow P, V$ form factors relevant for phenomenological applications.

A factorization formula for the heavy-light form factor of the form $f = C\zeta + \phi_B \otimes T \otimes \phi_M$ was first proposed in Ref. [16], based on the study of low order contributions in perturbation theory. The effective theory analysis in [17] establishes such a factorization theorem in the form of Eq. (8) to all orders in α_s , and at leading order in Λ/Q . It also shows that ζ depends on the scale $Q\Lambda$, as well as Λ^2 .

The two terms in the factorization formula Eq. (8) are of the same order in $\lambda = \Lambda/Q$, such that their relative numerical contributions can be comparable. Their scaling can be obtained from a simple model independent power counting argument as follows [17]. The SCET_I operators in Eq. (7) scale like λ^3 . After matching onto SCET_{II} the scaling of the collinear fields gets an additional λ . This gives for the total scaling of each of the two terms in Eq. (8) $\lambda^3 \times \lambda \times \lambda^{-1} \times \lambda^{-3/2} \sim \Lambda^{3/2}$, where the factors of λ^{-1} and $\lambda^{-3/2}$ correspond to the scaling of the light meson and B states, respectively.

Although the numerical values of the soft form factors ζ are not constrained by the effective theory, and have to be obtained from model computations or lattice QCD, the factorization results have significant predictive power. For example, using as input the form factor $f_+(q^2)$ as measured in $B \rightarrow \pi e \nu$, the remaining $B \rightarrow \pi$ form factors can be computed using Eqs. (16)-(18) and $\phi_B(k_+), \phi_\pi(x)$. Finally, the explicit results in Eqs. (16)-(21) can be used to calculate Sudakov effects from the RG running of the Wilson coefficients C_i, B_i . It will be interesting to see how the results of this running compare with the results in Refs. [15, 22, 18, 19].

The factorization relations have to be extended for the case of the penguin mediated rare radiative decays such as $B \rightarrow V\gamma$ and $B \rightarrow V\ell^+\ell^-$ ($V = K^*, \rho$), to account for the contributions of weak 4-quark operators. These effects have been computed in Refs. [24, 25] and contribute about 5-10% to the observed branching ratios.

3 Zero recoil region

In the low recoil region for the final meson, corresponding to maximal $q^2 \sim (m_B - m_M)^2$, heavy quark symmetry can be applied to describe the heavy-light form factors. For a heavy final meson $B \rightarrow D^{(*)}\ell\nu$, the normalization is fixed from the symmetry, with the leading power corrections of order Λ/m_b vanishing for certain form factors [27]. No such information is available for light

final mesons, although several properties of the heavy-to-light form factors can be established in a model-independent way.

The heavy mass scaling of the form factors can be straightforwardly derived from the mass dependence of the $|B\rangle$ states implicit in their relativistic normalization $|\bar{B}(p)\rangle \sim \sqrt{m_b}$. Adopting the usual definition of the formfactors (see, e.g. [16]), one finds the scaling laws [26]

$$T_1 + \frac{m_B^2 - m_V^2}{q^2}(T_1 - T_2) \propto m_b^{1/2}, \quad T_1 - \frac{m_B^2 - m_V^2}{q^2}(T_1 - T_2) \propto m_b^{-1/2} \quad (9)$$

$$V(q^2) \propto m_b^{1/2}, \quad A_1(q^2) \propto m_b^{-1/2}.$$

Relations among form factors of different currents can also be derived. There are three such relations for a vector light meson, and one relation for the pseudoscalar meson. For example, two of the $B \rightarrow V$ relations are [26]

$$T_1 + \frac{m_B^2 - m_V^2}{q^2}(T_1 - T_2) = \frac{2m_B}{m_B + m_V}V(q^2) + O(m_b^{-1/2}) \quad (10)$$

$$T_1 - \frac{m_B^2 - m_V^2}{q^2}(T_1 - T_2) = -\frac{2E}{m_B + m_V}V(q^2) + \frac{m_B + m_V}{m_B}A_1(q^2) + O(m_b^{-3/2}). \quad (11)$$

These relations are relevant for a method discussed in Refs. [28, 29] for determining the CKM matrix element $|V_{ub}|$ from exclusive B decays. This method combines data on semileptonic $B \rightarrow \rho \ell \nu$ and rare radiative decays $B \rightarrow K^* \ell^+ \ell^-$ near the zero recoil point, and $|V_{ub}|$ is extracted from the ratio [28, 29]

$$\frac{d\Gamma(B \rightarrow \rho e \nu)/dq^2}{d\Gamma(B \rightarrow K^* \ell^+ \ell^-)/dq^2} = \frac{8\pi^2}{\alpha^2} \frac{|V_{ub}|^2}{|V_{tb}V_{ts}^*|^2} \frac{1}{|C_9|^2 + |C_{10}|^2} \frac{|A_1^{B \rightarrow \rho}(q^2)|^2}{|A_1^{B \rightarrow K^*}(q^2)|^2} \frac{(m_B + m_\rho)^2}{(m_B + m_{K^*})^2} \frac{1}{1 + \Delta(q^2)} \quad (12)$$

The SU(3) breaking in this result can be eliminated using a Grinstein-type double ratio [30] and data on semileptonic $D \rightarrow K^*(\rho) e \bar{\nu}$ decays, resulting in a $|V_{ub}|$ determination at the 10% level [29].

Recently, the leading power corrections to the heavy quark symmetry relations Eqs. (10), (11) have been computed in Ref. [31]. Contrary to naive expectations, they have a very simple form and depend only on the form factors of the dimension-4 currents $\bar{q}iD_\mu(\gamma_5)b$. These corrections are required for example to determine the tensor form factor $T_1(q^2)$ in terms of V, A_1 measured in exclusive semileptonic $B \rightarrow V \ell \nu$ decays. It is easy to see that combining the symmetry relations Eqs. (10), (11) in order to extract T_1 is possible only if the leading correction of $O(m_b^{-1/2})$ to Eq. (10) is known (since the latter is of the same order as the terms shown on the RHS of Eq. (11)).

We give here an alternative derivation of this relation, and generalize it beyond the low recoil assumption implicit in the HQET derivation in [31]. Using the QCD equation of motion for the quark fields one finds

$$i\partial^\nu(\bar{q}i\sigma_{\mu\nu}b) = -(m_b + m_q)\bar{q}\gamma_\mu b + 2\bar{q}iD_\mu b - i\partial_\mu(\bar{q}b). \quad (13)$$

Taking the $B \rightarrow V$ matrix element of this relation one finds

$$T_1(q^2) = \frac{m_b + m_q}{m_B + m_V}V(q^2) - \mathcal{D}(q^2) \rightarrow \begin{cases} \frac{m_B - \bar{A}}{m_B + m_V}V(q^2) - \mathcal{D}(q^2) + O(m_b^{-3/2}) & \text{(low recoil)} \\ V(q^2) - \mathcal{D}(q^2) + O(Q^{-5/2}) & \text{(large energy)} \end{cases} \quad (14)$$

The first equality is exact and holds for arbitrary recoil, while the second relation gives its asymptotic form in the low recoil and large energy regions, respectively. The form factor $\mathcal{D}(q^2)$ (scaling like $\mathcal{D} \propto m_b^{-1/2}$ in the low recoil region) is defined as

$$\langle V(p', \varepsilon) | \bar{q} i D_\mu b | \bar{B}(p) \rangle = i \mathcal{D}(q^2) \epsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} p^\rho p'^\sigma, \quad (15)$$

and vanishes exactly in the constituent quark model [31], which suggests that its value could be small. It would be interesting to see if this suppression is confirmed by nonperturbative methods such as QCD sum rules or lattice QCD.

Similar results are obtained in Ref. [31] for subleading power corrections to all the other $B \rightarrow P, V$ form factor relations in the zero-recoil region. In all these cases the subleading terms depend only on form factors of the local dimension-4 operators $\bar{q} i D_\mu (\gamma_5) b$. These results were used in Ref. [32] to estimate the subleading corrections of $O(\Lambda/m_b)$ to the $|V_{ub}|$ determination using Eq. (12). These corrections can be as large as 5%, and are dominated by one of the (unknown) form factors of $\bar{q} i D_\mu \gamma_5 b$. Quark model estimates of this matrix element suggest that the correction is under a few percent, and more precise determinations (lattice QCD) could help to reduce or eliminate this source of uncertainty.

Note added. The convergence of the k_+ convolution in Eq. (8) was shown in [33], and very recently further work on heavy-light form factors within SCET studying the convergence of the convolution integrals was reported in [34].

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Appendix

We present here the complete results for the form factors in factorized form. Adopting the usual parameterization of the form factors, the explicit results for the $B \rightarrow P$ form factors are [10] (we use here the notations of [10] for the Wilson coefficients of SCET operators)

$$f_+(q^2) = \left(C_1^{(v)} + \frac{E}{m_B} C_2^{(v)} + C_3^{(v)} \right) \zeta^P \quad (16)$$

$$+ N_0 \int dx dl_+ \left\{ \frac{2E - m_B}{m_B} \left[B_1^{(v)} - \frac{E}{m_B - 2E} B_2^{(v)} - \frac{m_B}{m_B - 2E} B_3^{(v)} \right] J_a \right. \\ \left. + \frac{2E}{m_b} \left[B_{11}^{(v)} - \frac{E}{m_B} B_{12}^{(v)} - B_{13}^{(v)} \right] J_b \right\} \phi_\pi(x) \phi_B^+(l_+)$$

$$\frac{m_B}{2E} f_0(q^2) = \left(C_1^{(v)} + \frac{m_B - E}{m_B} C_2^{(v)} + C_3^{(v)} \right) \zeta^P \quad (17)$$

$$+ N_0 \int dx dl_+ \left\{ \frac{m_B - 2E}{m_B} \left[B_1 + \frac{m_B - E}{m_B - 2E} B_2^{(v)} + \frac{m_B}{m_B - 2E} B_3^{(v)} \right] J_a \right.$$

$$\begin{aligned}
& + \frac{2E}{m_b} \left[B_{11}^{(v)} - \frac{m_B - E}{m_B} B_{12}^{(v)} - B_{13}^{(v)} \right] J_b \Big\} \phi_\pi(x) \phi_B^+(l_+) \\
\frac{m_B}{m_B + m_P} f_T(q^2) &= \left(C_1^{(t)} - C_2^{(t)} - C_4^{(t)} \right) \zeta^P \\
& + N_0 \int_0^1 dx dl_+ \left\{ - \left[B_1^{(t)} - B_2^{(t)} - 2B_3^{(t)} + B_4^{(t)} \right] J_a - \frac{2E}{m_b} [B_{15}^{(t)} + B_{16}^{(t)} - B_{18}^{(t)}] J_b \right\} \phi_B^+(l_+) \phi(x),
\end{aligned} \tag{18}$$

with $N_0 = f_B f_P m_B / (4E^2)$.

The corresponding results for the $B \rightarrow V$ form factors read

$$\begin{aligned}
\frac{m_B}{m_B + m_V} V(q^2) &= C_1^{(v)} \zeta_\perp^V - N_\perp \int_0^1 dx dl_+ \left[-\frac{1}{2} B_4^{(v)} J_a^\perp + \frac{E}{m_b} (2B_{11}^{(v)} + B_{14}^{(v)}) J_b^\perp \right] \phi_B^+(l_+) \phi_\perp(x) \\
\frac{m_B + m_V}{2E} A_1(q^2) &= C_1^{(a)} \zeta_\perp^V - N_\perp \int_0^1 dx dl_+ \left[-\frac{1}{2} B_4^{(a)} J_a^\perp + \frac{E}{m_b} (2B_{11}^{(a)} + B_{14}^{(a)}) J_b^\perp \right] \phi_B^+(l_+) \phi_\perp(x) \\
A_0(q^2) &= \left(C_1^{(a)} + \frac{m_B - E}{m_B} C_2^{(a)} + C_3^{(a)} \right) \zeta_\parallel^V
\end{aligned} \tag{19}$$

$$\begin{aligned}
& + N_\parallel \int_0^1 dx dl_+ \left\{ \left[\frac{m_B - 2E}{m_B} B_1^{(a)} + \frac{m_B - E}{m_B} B_2^{(a)} + B_3^{(a)} \right] J_a \right. \\
& \quad \left. - \frac{2E}{m_b} \left[-B_{11}^{(a)} + \frac{m_B - E}{m_B} B_{12}^{(a)} + B_{13}^{(a)} \right] J_b \right\} \phi_B^+(l_+) \phi_\parallel(x) \\
\frac{m_B E}{m_B + m_V} A_2(q^2) - \frac{1}{2} (m_B + m_V) A_1(q^2) &= - \left(C_1^{(a)} + \frac{E}{m_B} C_2^{(a)} + C_3^{(a)} \right) m_V \zeta_\parallel^V
\end{aligned} \tag{20}$$

$$\begin{aligned}
& + m_V N_\parallel \int_0^1 dx dl_+ \left\{ \left[\frac{m_B - 2E}{m_B} B_1^{(a)} - \frac{E}{m_B} B_2^{(a)} - B_3^{(a)} \right] J_a \right. \\
& \quad \left. - \frac{2E}{m_b} \left[B_{11}^{(a)} - \frac{E}{m_B} B_{12}^{(a)} - B_{13}^{(a)} \right] J_b \right\} \phi_B^+(l_+) \phi_\parallel(x) \\
T_1(q^2) = \frac{m_B}{2E} T_2(q^2) &= \left\{ C_1^{(t)} - \frac{m_B - E}{m_B} C_2^{(t)} - C_3^{(t)} \right\} \zeta_\perp^V
\end{aligned} \tag{21}$$

$$\begin{aligned}
& - \frac{1}{2} N_\perp \int_0^1 dx dl_+ \left\{ \left[B_5^{(t)} + \frac{m_B - E}{m_B} B_6^{(t)} \right] J_a^\perp \right. \\
& \quad \left. - \frac{2E}{m_b} \left[2B_{15}^{(t)} + 2B_{17}^{(t)} + B_{19}^{(t)} + B_{21}^{(t)} + \frac{m_B - E}{m_B} (2B_{16}^{(t)} + B_{20}^{(t)}) \right] J_b^\perp \right\} \phi_B^+(l_+) \phi_\perp(x) \\
ET_3(q^2) - \frac{m_B}{2} T_2(q^2) &= -(C_1^{(t)} - C_2^{(t)} - C_4^{(t)}) m_V \zeta_\parallel^V
\end{aligned} \tag{22}$$

$$+ m_V N_\parallel \int_0^1 dx dl_+ \left\{ \left[B_1^{(t)} - B_2^{(t)} - 2B_3^{(t)} - B_4^{(t)} \right] J_a + \frac{2E}{m_b} (B_{15}^{(t)} + B_{16}^{(t)} - B_{18}^{(t)}) J_b \right\} \phi_B^+(l_+) \phi_\parallel(x)$$

where $N_\perp = m_B / (4E^2) f_B f_V^T$ and $N_\parallel = m_B / (4E^2) f_B f_V$. There are 2 jet functions $J_{a,b}$ contributing to $B \rightarrow P, V_\parallel$ (defined as in [10]), and 2 other jet functions contributing only to $B \rightarrow V_\perp$, denoted as $J_{a,b}^\perp$. At tree level they are equal $J_{a,b}^{(\perp)}(z, x, l_+) = \frac{\pi \alpha_s C_F}{N_c} \frac{1}{\bar{x} l_+}$, but in general they are different. The Wilson coefficients satisfy $C_{1-3}^{(v)} = C_{1-3}^{(a)}$ and $B_{1-4}^{(v)} = B_{1-4}^{(a)}$ in the NDR scheme. Reparameterization invariance constrains them as $B_{1-3}^{(v,a,t)} = C_{1-3}^{(v,a,t)}$, $B_4^{(v,a)} = -2C_3^{(v,a)}$, $B_4^{(t)} = C_4^{(t)}$, $B_5^{(t)} = 2C_3^{(t)}$, $B_6^{(t)} = -2C_4^{(t)}$ [8, 10]. At tree level they are given by $C_1^{(v,a,t)} = 1$, $B_1^{(v,a,t)} = 1$, $B_{13}^{(v,a)} = -1$, $B_{17}^{(t)} = 1$.

References

- [1] A. Ali, P. Ball, L. T. Handoko and G. Hiller, Phys. Rev. D **61**, 074024 (2000); A. Ali, E. Lunghi, C. Greub and G. Hiller, Phys. Rev. D **66**, 034002 (2002); A. Ali and E. Lunghi, Eur. Phys. J. C **21**, 683 (2001); G. Hiller, *Phenomenology of new physics*, in these proceedings.
- [2] B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0207080; S. B. Athar *et al.* [CLEO Collaboration], arXiv:hep-ex/0304019; B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0308042; K. Abe [Belle Collaboration], arXiv:hep-ex/0308044; E. Thorndike, *Inclusive and exclusive V_{ub} measurements*, in these proceedings; A. Ishikawa, *Review of $b \rightarrow s\ell^+\ell^-$ and $B \rightarrow \ell^+\ell^-$ decays*, in these proceedings.
- [3] P. Ball, arXiv:hep-ph/0306251; P. Ball, arXiv:hep-ph/0308249.
- [4] D. Becirevic, *Lattice QCD for B decays*, in these proceedings.
- [5] C. W. Bauer, S. Fleming and M. E. Luke, Phys. Rev. D **63**, 014006 (2001); C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001); C. W. Bauer and I. W. Stewart, Phys. Lett. B **516**, 134 (2001).
- [6] S. Fleming, *The Soft Collinear Effective Theory*, in these proceedings.
- [7] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **65**, 054022 (2002)
- [8] J. Chay and C. Kim, Phys. Rev. D **65**, 114016 (2002); A. V. Manohar, T. Mehen, D. Pirjol and I. W. Stewart, Phys. Lett. B **539**, 59 (2002).
- [9] M. Beneke, A. P. Chapovsky, M. Diehl and T. Feldmann, Nucl. Phys. B **643**, 431 (2002); M. Beneke and T. Feldmann, Phys. Lett. B **553**, 267 (2003).
- [10] D. Pirjol and I. W. Stewart, Phys. Rev. D **67**, 094005 (2003).
- [11] E. Lunghi, D. Pirjol and D. Wyler, Nucl. Phys. B **649**, 349 (2003).
- [12] G. P. Korchemsky, D. Pirjol and T. M. Yan, Phys. Rev. D **61**, 114510 (2000); S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. B **650**, 356 (2003) S. W. Bosch, R. J. Hill, B. O. Lange and M. Neubert, Phys. Rev. D **67**, 094014 (2003).
- [13] S. Descotes-Genon and C. T. Sachrajda, Phys. Lett. B **557**, 213 (2003).
- [14] S. J. Brodsky and P. Lepage, in *Perturbative Quantum Chromodynamics* (World Scientific, Singapore, 1989), pp. 93-240.
- [15] R. Akhouri, G. Sterman and Y. P. Yao, Phys. Rev. D **50**, 358 (1994).
- [16] M. Beneke and T. Feldmann, Nucl. Phys. B **592**, 3 (2001)
- [17] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **67**, 071502 (2003).

- [18] T. Kurimoto, H. n. Li and A. I. Sanda, Phys. Rev. D **65**, 014007 (2002).
- [19] H. n. Li, arXiv:hep-ph/0303116.
- [20] C. W. Bauer, D. Pirjol and I. W. Stewart, arXiv:hep-ph/0303156.
- [21] J. Charles et al., Phys. Rev. D **60**, 014001 (1999).
- [22] S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. B **625**, 239 (2002).
- [23] R. J. Hill and M. Neubert, Nucl. Phys. B **657**, 229 (2003)
- [24] A. J. Buras, A. Czarnecki, M. Misiak and J. Urban, Nucl. Phys. B **631**, 219 (2002);
M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. B **612**, 25 (2001); S. W. Bosch and
G. Buchalla, Nucl. Phys. B **621**, 459 (2002); A. Ali and A. Y. Parkhomenko, Eur. Phys.
J. C **23**, 89 (2002); J. g. Chay and C. Kim, arXiv:hep-ph/0305033.
- [25] B. Grinstein and D. Pirjol, Phys. Rev. D **62**, 093002 (2000); A. Hardmeier, E. Lunghi,
D. Pirjol and D. Wyler, arXiv:hep-ph/0307171.
- [26] N. Isgur and M. B. Wise, Phys. Rev. D **42**, 2388 (1990).
- [27] N. Isgur and M. B. Wise, Phys. Lett. B **237**, 527 (1990); M. E. Luke, Phys. Lett. B **252**,
447 (1990).
- [28] A. I. Sanda and A. Yamada, Phys. Rev. Lett. **75**, 2807 (1995).
- [29] Z. Ligeti and M. B. Wise, Phys. Rev. D **53**, 4937 (1996); Z. Ligeti, I. W. Stewart and
M. B. Wise, Phys. Lett. B **420**, 359 (1998).
- [30] B. Grinstein, Phys. Rev. Lett. **71**, 3067 (1993).
- [31] B. Grinstein and D. Pirjol, Phys. Lett. B **533**, 8 (2002)
- [32] B. Grinstein and D. Pirjol, Phys. Lett. B **549**, 314 (2002).
- [33] B. O. Lange and M. Neubert, arXiv:hep-ph/0303082.
- [34] M. Beneke and Th. Feldmann, CERN report CERN-TH/2003-202, hep-ph/0308303.